

Present and future evidence for evolving dark energy

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We compute the Bayesian evidences for one- and two-parameter models of evolving dark energy, and compare them to the evidence for a cosmological constant, using current data from Type Ia supernova, baryon acoustic oscillations, and the cosmic microwave background. We use only distance information, ignoring dark energy perturbations. We find that, under various priors on the dark energy parameters, Λ CDM is currently favoured as compared to the dark energy models. We consider the parameter constraints that arise under Bayesian model averaging, and discuss the implication of our results for future dark energy projects seeking to detect dark energy evolution. The model selection approach complements and extends the figure-of-merit approach of the Dark Energy Task Force in assessing future experiments, and suggests a significantly-modified interpretation of that statistic.

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I. INTRODUCTION

A key challenge for cosmology is to uncover the nature of the force which is causing the Universe to expand at an accelerating rate today. The cause, dubbed dark energy, could be an unknown energy component with negative pressure [1], a modification of general relativity [2], or simply a cosmological constant. For reviews on the subject, see for example Ref. [3].

There are many planned and proposed dark energy experiments that aim to constrain dark energy parameters, using a combination of complementary techniques. These include the luminosity distance–redshift relation of Type Ia supernovae (SNe Ia), the angular-diameter distance–redshift and expansion rate–redshift relations measured by baryon acoustic oscillations (BAO), and use of weak gravitational lensing to probe the growth rate of structures. The cosmic microwave background (CMB) also provides a very useful handle on dark energy by pinning down the distance to the last-scattering surface, and also via the Integrated Sachs–Wolfe effect and by detecting clusters through the Sunyaev–Zel’dovich effect. Approaches to constraining dark energy were overviewed in the recent report of the DoE/NASA/NSF Dark Energy Task Force (DETF) [4].

A primary aim of future experiments is to distinguish evolving dark energy from a cosmological constant. When seeking to compare models, especially with different numbers of variable parameters, one should use the concepts of model selection rather than those of parameter estimation (e.g. Refs. [5, 6]). Model selection quantifies how well the data conform to the overall predictions of a model, which depends on model dimensionality and model priors. In addressing the primary goal, a satisfactory representation of many evolving dark energy models turns out to be an unknown energy component with equation of state $w(a) = w_0 + w_a(1 - a)$, where a is the scale factor. Simpler alternatives may be the constant w model with negative pressure, and the cos-

mological constant model with fixed $w = -1$. This is a natural area for the application of model selection statistics [7, 8], which we take up in this paper. Here we update and extend work by Saini et al. [9], who were the first to apply Bayesian model selection to dark energy models. For alternative views on determining the number of dark energy parameters, see Ref. [10].

We do not consider growth-of-structure constraints, which ultimately will be required to distinguish between dark energy and modified gravity models for the acceleration [4, 11]. In the phenomenological approach adopted here, the dynamical evolution of w could be attributed to either phenomenon. At present, the structure formation growth factor theory is known only for specific modified gravity models, and further development is needed before such models can be usefully considered in the model selection framework. In any case, at the present time these observations are not competitive with the ones we use.

In this paper we compute the Bayesian evidence for evolving dark energy versus that of a cosmological constant given current distance measurements from CMB, SNe Ia, and BAO data, ignoring dark energy perturbations. In light of this result, we discuss the probability that future experiments will detect evolving dark energy, and the implications of this in assessing the capabilities of future experiments.

II. METHOD AND MODELS

Bayesian model selection extends the usual parameter estimation framework by assigning probabilities to *sets* of parameters, known as models, as well as the usual probability distributions of parameter values for each specific choice of model. The key statistic of Bayesian model selection is the *Bayesian evidence* E , being the average likelihood of the model over its prior parameter ranges [12, 13]. This quantity updates the prior model probability to the posterior model probability, enabling one to

compare different models according to their probability.

The use of the Bayesian evidence has lagged behind parameter estimation techniques in the cosmology literature because of the difficulty in computing the required integral to high accuracy, so as to be able to distinguish between the models of interest. The nested sampling algorithm, proposed by Skilling [14] and implemented for cosmological applications by some of us in Ref. [7], has proven to be computationally efficient and accurate. It is a simple algorithm and more general than thermal methods as used in Refs. [15, 16]. For instance, nested sampling can handle multiphase problems, in which $\ln L$ is not a concave function of $\ln X$, where X is the cumulative probability mass within isolikelihood surfaces and L the likelihoods of the surfaces — thermal methods fail on such problems (John Skilling, private communication).

We use the nested sampling algorithm to compute the Bayesian evidences [7, 14, 17]. Our code, called CosmoNest, is available at the URL www.cosmonest.org. As compared to the public version, it was modified so that instead of using the power spectra for each model, it used the data and likelihoods described in the next section. As we are not computing power spectra the calculation proceeds very swiftly, taking just a few minutes to obtain multiple estimates of the evidence of a model. The estimates can then be combined into a mean evidence and an error on that mean.

We consider five different models in all, corresponding to different parametrizations of the equation of state w and/or different parameter priors. The basic models are Λ CDM ($w = -1$, Model I), a one-parameter model with constant w , and a two-parameter model $w(a) = w_0 + w_a(1 - a)$, where w_0 and w_a are constants. This last parametrization, introduced by Chevallier and Polarski [18], is a good approximation to many dark energy models, while the constant w model is purely phenomenological. In addition to the equation of state, each model requires two further parameters to complete its specification, the matter density Ω_m and the Hubble constant H_0 .

For the latter two parametrizations, we make two separate choices of prior in order to explore this dependence. For the constant w case these are $-1 \leq w \leq -0.33$ (Model II) and $-2 \leq w \leq -0.33$ (Model III), the former enforcing the weak energy condition and the latter allowing phantom models. For the two-parameter model, Model IV has flat priors of $-2 \leq w_0 \leq -0.33$, $-1.33 \leq w_a \leq 1.33$ (the prior on w_a being particularly arbitrary), while Model V corresponds to the quintessence prior of $-1 \leq w(a) \leq 1$ imposed between $z = 0$ and 2.

III. OBSERVATIONAL DATA

We use data in a manner very similar to Wang and Mukherjee [19], which can be consulted for more details.

We use the CMB shift parameter measured by the three-year Wilkinson Microwave Anisotropy Probe

(WMAP) observations [20, 21], of $R = 1.70 \pm 0.03$ [19], which is mostly independent of assumptions made about dark energy. The shift parameter R is [22]

$$R \equiv \Omega_m^{1/2} \int_0^{z_{\text{CMB}}} \frac{dz'}{E(z')}, \quad (1)$$

where z_{CMB} is the redshift of recombination. In a flat Universe

$$E(z) \equiv \left[\Omega_m(1+z)^3 + (1-\Omega_m) \frac{\rho_X(z)}{\rho_X(0)} \right]^{1/2}, \quad (2)$$

with ρ_X denoting the dark energy density given by

$$\frac{\rho_X(z)}{\rho_X(0)} = \exp \left\{ \int_0^z dz' \frac{3[1+w(z')]}{1+z'} \right\}. \quad (3)$$

In that case $R = (\Omega_m H_0^2)^{1/2} r(z_{\text{CMB}})/c$, and is well determined as both $\Omega_m h^2$ and $r(z_{\text{CMB}})$ are accurately measured by CMB data.

We use the BAO measurement from the Sloan Digital Sky Survey (SDSS) luminous red galaxies, $d_V(0.35) = 1.300 \pm 0.088 \text{ Gpc}$ [23], obtained from power spectrum estimates and consistent with the result of Ref. [24] obtained using the estimated correlation function. Here the distance parameter is

$$d_V(z_{\text{BAO}}) = \left[r^2(z_{\text{BAO}}) \frac{cz_{\text{BAO}}}{H(z_{\text{BAO}})} \right]^{1/3}, \quad (4)$$

where $r(z)$ is the comoving distance, and $H(z)$ is the Hubble parameter. For the SDSS luminous red galaxies, the mean survey redshift is $z_{\text{BAO}} = 0.35$.¹

We use SN Ia data from the HST/GOODS programme [25] (Riess04) and the first year Supernova Legacy Survey [26] (Astier05), together with nearby SN Ia data. The comparison of results from these two SN Ia datasets provides a consistency check. We do not combine the two SN Ia datasets, as they have systematic differences in data processing; see the discussion in Ref. [19].

We use the Riess04 ‘gold’ sample flux-averaged with $\Delta z = 0.05$. This sample includes 9 SNe Ia at $z > 1$, and appears to have systematic effects from weak lensing, or another effect that mimics weak lensing qualitatively. This would bias the distance estimates somewhat without flux averaging [27, 28], and so we use it on these SNe [29].

We have also added a conservative estimate of the intrinsic dispersion of SN Ia peak brightness, 0.15 mag, in quadrature with the distance moduli of Astier05, rather than the smaller intrinsic dispersion derived by them by requiring a reduced $\chi^2 = 1$ in their model fitting. This is

¹ The SDSS BAO result has been computed for a scalar spectral index value of $n_s = 0.98$, and should be scaled by $(n_s/0.98)^{-0.35}$ [24] for a different ‘best-fit’ n_s . For $n_s \simeq 0.95$ following WMAP3 [21] this is an insignificant factor, which however we do include.

TABLE I: The mean $\Delta \ln E$ relative to the Λ CDM model together with its uncertainty, the information content H , the minimum χ^2 , and the parameter constraints, for each of the models considered and for each of two data combinations. Uncertainties on H_0 are statistical only, and do not include systematic uncertainties. The models differ by virtue of the number of free parameters, here in the dark energy sector, and/or the priors on those parameters. For reference, $\ln E$ for the Λ CDM model was found to be -20.1 ± 0.1 for the compilation with Riess04 and -52.3 ± 0.1 for that with Astier05.

data used	Model			
WMAP+SDSS+	$\Delta \ln E$	H	χ^2_{\min}	parameter constraints
	Model I: Λ			
Riess04	0.0	5.7	30.5	$\Omega_m = 0.26 \pm 0.03$, $H_0 = 65.5 \pm 1.0$
Astier05	0.0	6.5	94.5	$\Omega_m = 0.25 \pm 0.03$, $H_0 = 70.3 \pm 1.0$
	Model II: constant w , flat prior $-1 \leq w \leq -0.33$			
Riess04	-0.1 ± 0.1	6.4	28.6	$\Omega_m = 0.27 \pm 0.04$, $H_0 = 64.0 \pm 1.4$, $w < -0.81$, -0.70^a
Astier05	-1.3 ± 0.1	8.0	93.3	$\Omega_m = 0.24 \pm 0.03$, $H_0 = 69.8 \pm 1.0$, $w < -0.90$, -0.83^a
	Model III: constant w , flat prior $-2 \leq w \leq -0.33$			
Riess04	-1.0 ± 0.1	7.3	28.6	$\Omega_m = 0.27 \pm 0.04$, $H_0 = 64.0 \pm 1.5$, $w = -0.87 \pm 0.1$
Astier05	-1.8 ± 0.1	8.2	93.3	$\Omega_m = 0.25 \pm 0.03$, $H_0 = 70.0 \pm 1.0$, $w = -0.96 \pm 0.08$
	Model IV: w_0 - w_a , flat prior $-2 \leq w_0 \leq -0.33$, $-1.33 \leq w_a \leq 1.33$			
Riess04	-1.1 ± 0.1	7.2	28.5	$\Omega_m = 0.27 \pm 0.04$, $H_0 = 64.1 \pm 1.5$, $w_0 = -0.83 \pm 0.20$, $w_a = --^b$
Astier05	-2.0 ± 0.1	8.2	93.3	$\Omega_m = 0.25 \pm 0.03$, $H_0 = 70.0 \pm 1.0$, $w_0 = -0.97 \pm 0.18$, $w_a = --^b$
	Model V: w_0 - w_a , $-1 \leq w(a) \leq 1$ for $0 \leq z \leq 2$			
Riess04	-2.4 ± 0.1	9.1	28.5	$\Omega_m = 0.28 \pm 0.04$, $H_0 = 63.6 \pm 1.3$, $w_0 < -0.78$, -0.60^a , $w_a = -0.07 \pm 0.34$
Astier05	-4.1 ± 0.1	11.1	93.3	$\Omega_m = 0.24 \pm 0.03$, $H_0 = 69.5 \pm 1.0$, $w_0 < -0.90$, -0.80^a , $w_a = 0.12 \pm 0.22$

^bWhere constraints on w are shown as upper limits only, the values are 68% and 95% marginalized confidence limits.

^c w_a is unconstrained in Model IV.

because the intrinsic dispersion in SN Ia peak brightness should be derived from the distribution of nearby SNe Ia, or SNe Ia from the same small redshift interval if the distribution in the peak brightness evolves with cosmic time. This distribution is not well known at present, but will become better known as more SNe Ia are observed by the nearby SN Ia factory [30]. By using the larger intrinsic dispersion, we allow some reasonable margin for the uncertainties in the SN Ia peak brightness distribution.

IV. RESULTS

We calculate the Bayesian evidence as our primary model selection statistic. We also calculate the information content H of the datasets, the best-fit χ^2 values, and the posterior parameter distributions within each model. Our main focus is on the evidence and the parameter distributions. All of these quantities are by-products of running CosmoNest to evaluate the evidence of a model [17].

A. Bayesian evidence E

The interpretational scale introduced by Jeffreys [31] defines a difference in $\ln E$ of greater than 1 as significant,

greater than 2.5 as strong, and greater than 5 as decisive, evidence in favour of the model with greater evidence.

Our results are summarized in Table I. The priors on the equation of state parameters were given earlier and are indicated in the table. Priors on the additional parameters are $0.1 \leq \Omega_m \leq 0.5$ and $40 \leq H_0 \leq 90$. For each model and data compilation we tabulate $\Delta \ln E$, which is the difference between the mean $\ln E$ of the Λ CDM model and the model concerned, plus the error on that difference, obtained from 8 estimates of the evidence of each model. Thus the Λ CDM entry is zero by definition.

We find that the WMAP+SDSS(BAO)+Astier05 data combination distinguishes amongst the models more strongly than does WMAP+SDSS(BAO)+Riess04 data, while showing the same general trends. Subsequently, our discussion uses Astier05 throughout.

Overall, the Λ CDM model (Model I) is a simple model that continues to give a good fit to the data. It is therefore rewarded for its predictiveness with the largest evidence, and remains the favoured model as found with an earlier dataset (of SNe alone) by Saini et al. [9]. The other models all show smaller evidences, though none are yet decisively ruled out. Nevertheless, there is distinct evidence against the two-parameter models, especially from the compilation including Astier05. Model V has a wider parameter range than Model IV and fares the worst, re-

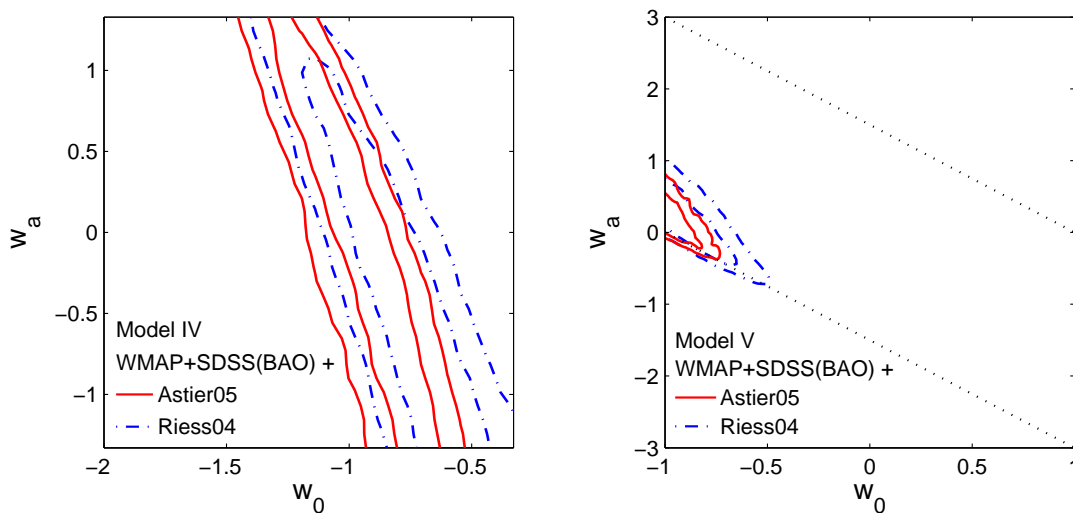


FIG. 1: 68% and 95% confidence constraints on w_0 and w_a for model IV (left plot) and model V (right plot). Note the axis ranges are different, to show the full prior ranges in each case. Model IV corresponds to a flat prior on the parameters over the range plotted, and Model V corresponds to a quintessence prior which amounts to a flat prior within the region shown by dotted lines (the contours go just a little out of that region due to the effect of binning the likelihoods of the obtained samples on a grid). The solid contours are for WMAP+SDSS(BAO)+Astier05, and dot-dashed contours are for WMAP+SDSS(BAO)+Riess04.

ceiving a large penalty for its lack of predictiveness of the data. The one-parameter models lie somewhere in between.

We interpret these results in the following section.

B. The information H

The information content of the data H is defined as minus the logarithm of the amount by which the posterior is compressed inside the prior by the data. We compute it from the posterior samples generated using nested sampling [17], and tabulate the values. H gives some indication of how many parameters a data set can support, as usually $H \approx N \log(\text{signal}/\text{noise})$ where N is the number of parameters [14]. If H changes significantly when new parameters are added, then that implies that the data have the potential to constrain the additional parameters effectively, and therefore have something conclusive to say about the distinction between the two models via the evidence. H is similar to the effective complexity of a model, as discussed in Ref. [32]. By definition, H depends on the prior, and the higher H is, the better the posterior is confined with respect to the prior.

C. Best-fit χ^2

The best-fit χ^2 obtained for each data set is listed in Table I, mainly for reference only. They were obtained from the highest-likelihood point found by the nested sampling algorithm. This will be close to, though not precisely at, the maximum, because the stopping crite-

rion for the nested sampling algorithm has to do with the convergence of the integral that estimates the evidence; the algorithm is not directly searching for the maximum-likelihood point. A naive model selection test, formalized as the likelihood ratio test, compares the difference in these best-fit values to the difference in number of model parameters. This does not however have a probabilistic interpretation, as the probability of the model is a property of its entire parameter range, not simply its best-fit values [12]. It ignores parameter priors and correlations.

Nevertheless, the lack of any significant improvement in χ^2_{\min} when going from the constant w models to the w_0 - w_a models could be used to conclude that the dataset is not interested in going to the two-parameter model.

D. Posterior parameter distributions

Parameter constraints for each of the models, obtained as described in Ref. [17] from the same samples that were used to compute the evidence, are tabulated in the final column of Table I. Likelihood contours for the dark energy parameters in Models IV and V are shown in Fig. 1, the contours in Model V being significantly cut off by the prior. 1D marginalized parameter constraints are shown in Fig. 2.

V. DISCUSSION: THE PRESENT PICTURE

The above results have quantified the impact of current data in constraining the models we have selected for investigation. There are considerable modelling uncer-

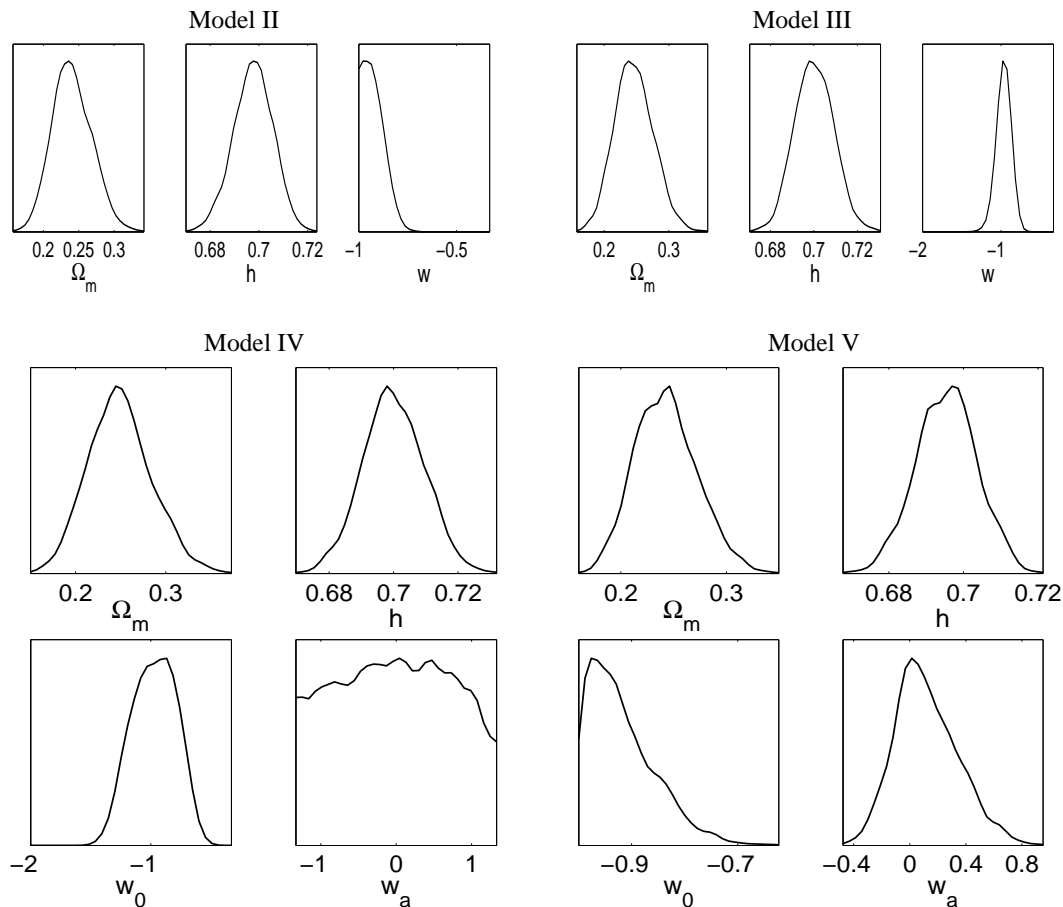


FIG. 2: Marginalized posterior parameter distributions in Models II, III, IV, and V, using the WMAP+SDSS(BAO)+Astier05 data combination.

tainties, both in the choice of parameter priors for each model, and in assigning prior model probabilities. For the latter, we have chosen to take them as equal, but anyone who thinks otherwise can readily account for it; regardless of what someone thinks about two models before looking at the data (the prior model probabilities), the evidence unambiguously states how that view is changed by the data. For the parameter priors, we have analyzed two choices for each model dimensionality to investigate the extent of the dependence.

In analyzing data in a situation where the correct choice of model is unknown, these uncertainties are unavoidable, but one can nevertheless use the Bayesian framework to draw conclusions. Within the model selection viewpoint, one should first ask of the status of the various models under discussion, and only then move on to consider parameter constraints.

A. Models

In comparing the model evidences, there are different ways to proceed. First, we can consider all five models as independent, so that converting the $\Delta \ln E$ into posterior

model probabilities, assuming equal prior model probabilities, gives 63%, 17%, 10%, 9%, and 1% for the five models respectively. Consequently, while we cannot say that any of the models is decisively ruled out, the balance of probability is currently tilted significantly in favour of Λ CDM, and the two-parameter equation of state model fares the worst.

Alternatively, we can consider each parametrization as representing a model, and within each parametrization marginalize over the different choices of prior that we considered plausible. In this approach we average the evidences (not their logarithms, as it is the evidences themselves which represent the model probability) to obtain $\Delta \ln E = -1.5$ for the constant w model and $\Delta \ln E = -2.6$ for the two-parameter model. The corresponding probabilities are then 77%, 18%, and 5% for Λ CDM, the one-parameter, and two-parameter dark energy models respectively. This approach gives quite similar results to the above, while avoiding penalizing the Λ CDM model for only having one choice of prior. However for the remainder of the paper we will not average over models in this way.

Finally, we might be interested only in a subset of the models; for instance, we may consider only the models

TABLE II: Parameter constraints from Bayesian model averaging using the WMAP+SDSS(BAO)+Astier05 data combination. Since the distributions of the dark energy parameters are generally nongaussian and/or asymmetric about the mean, their 68% and 95% marginalized limits are separately indicated. Some confidence limits for w_a are precisely zero due to the delta-function contribution from Models I, II, and III superimposed on the extended tails from Models IV and V.

models used	parameter constraints
all five models	$\Omega_m = 0.25 \pm 0.03$, $H_0 = 70.1 \pm 1.0$, $w_0 = -0.97^{+0.07, +0.19}_{-0.03, -0.20}$, $w_a = 0.0^{+0.0, +0.8}_{-0.0, -0.8}$
models I, II, & V	$\Omega_m = 0.24 \pm 0.03$, $H_0 = 70.1 \pm 1.0$, $w_0 < -0.98, -0.86$, $w_a = 0.0^{+0.0, +0.0}_{-0.0, -0.0}$

that do not allow $w < -1$ (models I, II, and V), motivated by quintessence models. Amongst these models the probability is divided as 78%, 21%, and 1% respectively.

Whichever the choice made, the overall conclusion is that the Λ CDM model is preferred by present data, but that there are non-negligible probabilities for the models of evolving dark energy. We will explore the implications of this for future dark energy searches in Section VI.

B. Parameter values and Bayesian model averaging

We now consider the implications of the model selection framework for constraints on the cosmological parameters. Within each model the usual parameter probability distribution analysis applies, as given in Section IV D. However we now need to combine these to derive parameter constraints that account for model uncertainty (the uncertainty in which model is the true model). The appropriate tool to carry this out is *Bayesian model averaging*, which is nicely summarized by Hoeting et al. [33].²

The basic idea is quite simple; rather than having a single probability distribution for a parameter, we instead have a superposition of its distributions in different models, weighted by the relative model probability. In some models the parameter may have a fixed value (e.g. $w = -1$ in Λ CDM), and then that component of the distribution is an appropriately-normalized delta-function. The set-up is analogous to quantum mechanics; whilst the true model is uncertain the distribution lies in a superposition of states, with the possibility that future measurements may collapse the probability into one of the models.

The posterior probabilities of the models are given via Bayes' theorem by

$$P(M_k|D) = \frac{P(D|M_k)P(M_k)}{\sum_k P(D|M_k)P(M_k)}, \quad (5)$$

² Bayesian model averaging has only been used once previously in cosmology, in interpreting simulated galaxy cluster data [34], and only very occasionally in astrophysics/geophysics [35]. A distinct idea, closely related to the themes of this article, is Bayesian survey design which averages an experimental figure of merit over a set of possible cosmological models [36].

where $P(D|M_k)$ is the evidence of model M_k . Here $P(M_k)$ are prior model probabilities, which we take to be equal across the models. Any other choice can be incorporated if required.

Within a gaussian approximation, it is easy to write down suitable expressions for model averaging the parameter means and variances [33], but it is practically as easy to manipulate the full distributions given by the parameter chains. One simply takes the chains from each model and weights them according to the model probability. That the elements will have noninteger weights is no problem (indeed CosmoNest chains, unlike those generated by Markov Chain Monte Carlo, already have noninteger weights with the weights within each chain summing to unity). All the chains can then be analyzed together by the usual means such as the `getdist` package of CosmoMC [37].

One shouldn't overstate the usefulness of this method, as the details depend on a lot of prior information: the precise choice of models, including their prior parameter ranges, to be averaged, and also the prior model probabilities. Nevertheless there are some general qualitative lessons to be learned.

The most important such lesson is that if one is seeking to limit a parameter around some special fiducial value, eg $w = -1$ for the equation of state, then the parameter errors are typically going to be *overestimated* if one ignores model uncertainty. The reason is that in the absence of a detection, a substantial part of the model probability is always going to be placed in the embedded model (in this case Λ CDM), which adds a delta-function to the probability distribution and hence suppresses the tails where the limits will be imposed.

The parameter constraints obtained from Bayesian model averaging the models together are summarized in Table II. Because of the presence of delta-functions in the averaged distribution, in some cases confidence limits can be precisely zero. The posterior distributions for the parameters are shown in Fig. 3, the left set of panels showing averaging over all five models, and the right set averaging the Λ model with the quintessence-type models (II and V).

The probability distributions of the parameters derived from current data, after taking into account model uncertainty by Bayesian model averaging over the models allowed by the data, summarize our current state of knowledge regarding these parameters. In this case we

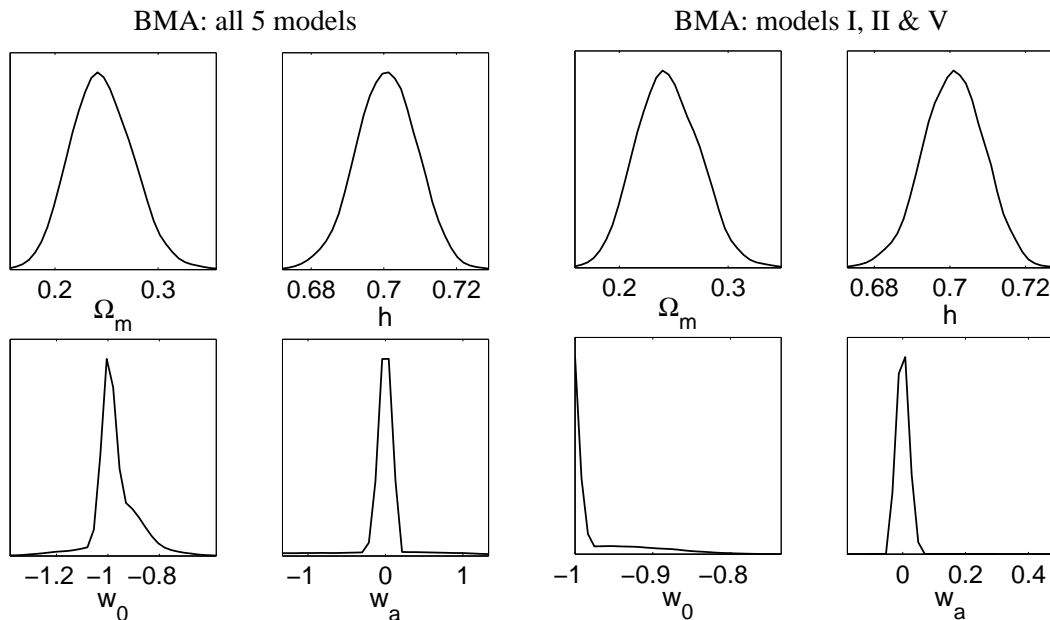


FIG. 3: Posterior parameter distributions obtained using the WMAP+SDSS(BAO)+Astier05 data combination from Bayesian model averaging (BMA). The left set of four panels averages over all the five models under consideration, and the right over the quintessence-type models (I, II and V) alone. Some smoothing of the delta-functions has been carried out by binning.

can see that even though many models are still allowed by the data, given the weight of the Λ CDM model, the constraints have tightened significantly around $w_0 = -1$ and $w_a = 0$. For instance, compare the model-averaged constraints of Fig. 3 (left) with the Model IV constraints in Fig. 2.

Note that Bayesian model averaging is an intrinsic part of the model selection framework, not an optional extra. As soon as one concedes that there might be different model descriptions of data to which probabilities should be assigned, consistent inference will then require those probabilities to be properly accounted in deriving parameter probability distributions. This is necessary too for consistent model selection forecasting, as we now describe.

VI. DISCUSSION: IMPLICATIONS FOR FUTURE SURVEYS

We now consider the implications of these results for future surveys, keeping this final discussion qualitative. In keeping with the previous section, an analysis of future data should first assess the validity of the various models being considered, and only then move on to parameter estimation. The model comparison may be decisive, leaving only one model on the table (which might be either Λ CDM or one of the evolving models), or it may still leave several viable models. If only one model survives then standard parameter estimation tools will be valid, otherwise model averaging should again be deployed to study parameter distributions.

The main aim of forecasting the power of future surveys is to enable informed choices as to which projects to fund. The Dark Energy Task Force (DETF) recently produced an influential report [4] quantifying the capabilities of a wide range of proposed experiments to constrain dark energy. Following ideas from Ref. [38], they defined a Figure-of-Merit (FoM) as the inverse of the area inside the 95% contour in the w_0 - w_a plane, for a fiducial Λ CDM model. Normalizing to present knowledge, this factor is typically a few to a few tens for proposed experiments of increasing sophistication.³

The DETF FoM presumes that the two-parameter dark energy model is the true one (i.e. that w_0 and w_a are parameters to be varied in fitting the data), and quantifies the extent to which future experiments will compress the allowed parameter range about the point $w_0 = -1$ and $w_a = 0$. What it does not do is allow for the possibility that the two-parameter model is not correct. To quote from the abstract of Ref. [33], “*Data analysts typically select a model from some class of models and then proceed as if the selected model had generated the data. This approach ignores the uncertainty in model selection, leading to inferences that are more risky than one thinks they are.*” One way to avoid this problem is to employ model selection forecasting, as described in Ref. [8], which proposed a FoM based on the parameter area in which Λ CDM cannot be strongly excluded using

³ These ideas have also been extended beyond survey comparison to the issue of survey design by Bassett and collaborators [36].

the Bayesian evidence.

We stress that the DETF FoM is a perfectly good way of distinguishing the capabilities of different experiments, even though it is a parameter estimation tool and those experiments are primarily seeking to answer model selection questions. It is entirely reasonable to believe that an experiment which is better at estimating parameters within a model will also be better at model selection of that model against embedded models. Our aim here is to advise caution against over-interpreting the DETF FoM, in terms of the probability that an upcoming experiment will actually detect dark energy evolution.

The model selection considerations we have outlined have three important implications in interpreting the DETF FoM.

1. *The chances of detecting dark energy evolution are much less than implied by the fractional shrinkage of parameter area.* For example, if the FoM says the area in the w_0 - w_a plane will shrink by a factor 10, this does not mean a 90% chance that evolution will be detected. This comment matches most people's intuition, but is quantified by the realization in model selection that a substantial part of the probability lies in the Λ CDM model. If this model is true, then obviously evolution cannot be detected as that would rule out the true model. Since present knowledge puts most of the probability in Λ CDM, as shown above, we can immediately conclude that the current chances of even an arbitrarily good experiment detecting dark energy evolution are less than half (with the significant caveat of the various model and parameter priors we have assumed).
2. *There is a substantial probability that Λ CDM is the correct model, but the DETF FoM does not quantify how well experiments will determine this.* If Λ CDM is the true model, then the outcome of future experiments will be to support that model. This too would be a highly-valuable outcome. In this case, there is another model selection based FoM, described in Ref. [8], which evaluates the strength with which an upcoming experiment is expected to deliver a model selection verdict *in favour* of Λ CDM under the assumption that that model is correct. Model selection approaches have the crucial property that, unlike parameter estimation methods, they can accrue positive support for the simpler model. As shown in Ref. [8], advanced experiments are capable of decisively ruling out the two-parameter models in favour of Λ CDM (see also Ref. [39]). This then is the answer to the often-asked question, how far do we have to tighten constraints on dark energy parameters before we can start to believe that Λ CDM is the true model. This question is often asked with parameter estimation forecasting techniques in mind, but the answer lies in model selection. A design goal of future experi-

ments should be that they are able to give a decisive verdict for Λ CDM if it is the true model.

3. *If evolution is neither detected nor decisively excluded, the DETF FoM will **overestimate** the parameter errors.* It overestimates because it does not incorporate Bayesian model averaging. A powerful experiment that fails to detect evolution is bound to push most of the model probability into the Λ CDM model, so that the eventual combined parameter chain includes only a small fraction of elements from the w_0 - w_a model. That is to say, the delta-function of probability at $w = -1$ will contain most of the posterior distribution. So an experiment which does not detect evolution will impose more powerful constraints than the FoM indicates.

None of the above affects the validity of the DETF FoM as a tool for quantifying the capabilities of different experiments, though one should bear in mind that it may prove inadequate if the true model is *more* complicated than the w_0 - w_a model [40]. Nevertheless, while the DETF FoM may correctly rank experiments relative to one another, since the principal goal of dark energy experiments is one of model selection, we would advocate where possible also analyzing their capabilities using model selection forecasting tools as described in Ref. [8] and this paper.

The approach we have outlined incorporates, modifies and extends the Expected Posterior Odds (ExPO) technique pioneered by Trotta [5]. This approach splits the model parameter space into regions where different model selection verdicts are expected, and then averages these over the current distribution in parameter space to obtain a probability of each outcome. Of course, only by actually doing the experiment do you discover which outcome does arise. Trotta did not however fully implement the model selection/Bayesian model averaging framework, as he computed the present probability distribution within one model only, whereas in Ref. [41] multiple models were included in an ExPO-type forecast. Ref. [8] extended ExPO to delineate parameter space regions where different model selection outcomes would be expected, and to define model selection figures of merit. The present paper further extends the framework to estimation of parameter uncertainties via Bayesian model averaging as well as calculation of model probabilities.

VII. CONCLUSIONS

We have carried out a model selection analysis of dark energy models, updating and expanding on an earlier analysis by Saini et al. [9]. We find, as did they, that the preferred model is the Λ CDM model, and indeed we find that the two-parameter w_0 - w_a model is quite significantly disfavoured already by present data.

We have made a first use of the concept of Bayesian model averaging [33] to obtain current cosmological pa-

parameter uncertainties. Bayesian model averaging generalizes the usual Bayesian parameter estimation methods to the situation where the choice of model is uncertain, and in the absence of detections typically significantly *strengthens* parameter constraints. Finally, we have described how to use this framework to project the probabilities of different outcomes to future dark energy experiments, and in particular to interpret the meaning of the figure-of-merit introduced by the Dark Energy Task Force [4].

We conclude that based on present knowledge the probability of future experiments detecting dark energy evolution is rather small, unless the various prior assumptions of our analysis prove to be ill-founded. This is simply because present data places the majority of the probability in the Λ CDM model. On the other hand, high-precision experiments may be able to decisively support

the Λ CDM model, this ability being measured by a model selection figure-of-merit given in Ref. [8]. If Λ CDM is not picked decisively, and neither is dark energy evolution detected, then they can give tighter limits on dark energy parameters than one would infer from the DETF figure-of-merit.

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